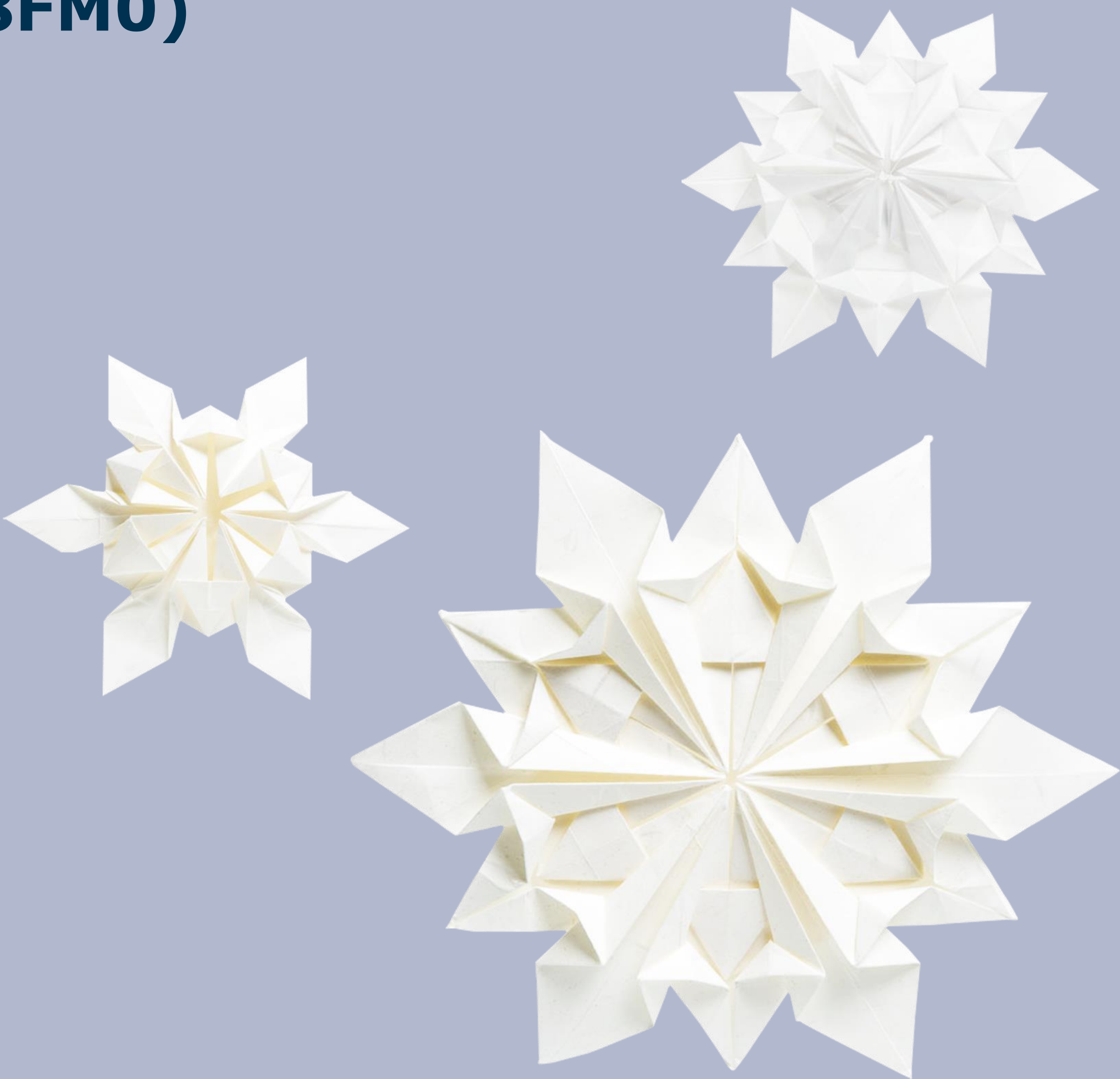
Pearson Edexcel Level 3 Advanced Subsidiary GCE in Further Mathematics (8FM0)



Sample Assessment Materials Model Answers – Core Pure Mathematics

First teaching from September 2017
First certification from June 2018

Sample Assessment Materials Model Answers – Core Pure Mathematics

Contents

Introduction

This booklet has been produced to support mathematics teachers delivering the new Pearson Edexcel Level 3 Advanced Subsidiary GCE in Mathematics (8FMO) specification for first teaching from September 2017.

This booklet looks at Sample Assessment Materials for AS Further Mathematics qualification, specifically at core pure mathematics questions, and is intended to offer model solutions with different methods explored.

Content of Core Pure Mathematics

Content	**AS level content**
Proof	Construct proofs using mathematical induction. Contexts include sums of series, divisibility and powers of matrices.
Complex numbers	Solve any quadratic equation with real coefficients. Solve cubic or quartic equations with real coefficients. Add, subtract, multiply and divide complex numbers in the form $x + \mathrm{i}y$ with x and y real. Understand and use the terms 'real part' and 'imaginary part'. Understand and use the complex conjugate. Know that non-real roots of polynomial equations with real coefficients occur in conjugate pairs. Use and interpret Argand diagrams. Convert between the Cartesian form and the modulus-argument form of a complex number. Knowledge of radians. Multiply and divide complex numbers in modulus argument form. Knowledge of radians and compound angle formulae is assumed. Construct and interpret simple loci in the argand diagram such as $z - a > r$ and arg $(z - a) = \theta$
Matrices	Add, subtract and multiply conformable matrices. Multiply a matrix by a scalar. Understand and use zero and identity matrices. Use matrices to represent linear transformations in 2-D. Successive transformations. Single transformations in 3-D. Find invariant points and lines for a linear transformation. Calculate determinants of: 2×2 and 3×3 matrices and interpret as scale factors, including the effect on orientation. Understand and use singular and non-singular matrices. Properties of inverse matrices. Calculate and use the inverse of non-singular 2×2 matrices and 3×3 matrices. Solve three linear simultaneous equations in three variables by use of the inverse matrix. Interpret geometrically the solution and failure of solution of three simultaneous linear equations.
Further algebra and functions	Understand and use the relationship between roots and coefficients of polynomial equations up to quartic equations.

Content	AS level content
	Form a polynomial equation whose roots are a linear transformation of the roots of a given polynomial equation (of at least cubic degree). Understand and use formulae for the sums of integers, squares and cubes and use these to sum other series.
Further calculus	Derive formulae for and calculate volumes of revolution.
Further vectors	Understand and use the vector and Cartesian forms of an equation of a straight line in 3-D. Understand and use the vector and Cartesian forms of the equation of a plane. Calculate the scalar product and use it to express the equation of a plane, and to calculate the angle between two lines, the angle between two planes and the angle between a line and a plane. Check whether vectors are perpendicular by using the scalar product. Find the intersection of a line and a plane. Calculate the perpendicular distance between two lines, from a point to a line and from a point to a plane.

Core Pure Mathematics 1

Question 1

$$f(z) = z^3 + pz^2 + qz - 15,$$

where p and q are real constants.

Given that the equation $f(z) = 0$ has roots

$$\alpha, \ \frac{5}{\alpha} \text{ and } \left(\alpha + \frac{5}{\alpha} - 1\right),$$

(a) solve completely the equation $f(z) = 0$.

(5)

$f(z) = z^3 + pz^2 + qz - 15$

Product of roots $= -(-15)$

$\alpha\left(\frac{5}{\alpha}\right)\left(\alpha + \frac{5}{\alpha} - 1\right) = 15$ M1

A1

$\alpha + \frac{5}{\alpha} - 1 = 3$

$\alpha^2 - 4\alpha + 5 = 0$

Complete the square — or use quadratic formula

$(\alpha - 2)^2 + 1 = 0$ — $\alpha = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$ M1

$\alpha = 2 \pm \mathrm{i}$ A1

If $\alpha = 2 + \mathrm{i}$, then $\frac{5}{\alpha} = 2 - \mathrm{i}$,

so $\alpha + \frac{5}{\alpha} - 1 = (2 + \mathrm{i}) + (2 - \mathrm{i}) - 1 = 3$

So roots of $f(z) = 0$ are $2 + \mathrm{i}$, $2 - \mathrm{i}$ and 3. A1

(b) Hence find the value of p.

(2)

p = –(sum of roots)
$p = -(2 + i + 2 - i + 3)$ M1
$p = -7$ A1

Alternative:

$z^3 + pz^2 + qz - 15 = (z - 3)(z^2 - 4z + 5)$
$= z^3 - 7z^2 + 17z - 15$ M1
so $p = -7$ A1

Question 2

The plane Π passes through the point A and is perpendicular to the vector **n**.

Given that

$$\overrightarrow{OA} = \begin{pmatrix} 5 \\ -3 \\ -4 \end{pmatrix} \text{ and } \mathbf{n} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix},$$

where O is the origin,

(a) find a Cartesian equation of Π.

(2)

Scalar product equation of a plane: **r.n** = **a.n**
r = general point
n = normal vector to plane
a = position vector of point A

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \bullet \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \\ -4 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$ M1

$3x - 1y + 2z = 15 + 3 - 8$
$3x - y + 2z = 10$ A1

With respect to the fixed origin O, the line l is given by the equation

$$\mathbf{r} = \begin{pmatrix} 7 \\ 3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -5 \\ 3 \end{pmatrix}.$$

The line l intersects the plane Π at the point X.

(b) Show that the acute angle between the plane Π and the line l is 21.2°, correct to one decimal place.

(4)

Scalar product formula: $\mathbf{n.d} = |\mathbf{n}||\mathbf{d}|\cos\theta$
$\mathbf{n}$ = normal vector to plane
$\mathbf{d}$ = direction vector of line

$\mathbf{n.d} = -3 + 5 + 6 = 8$	B1
$\lvert\mathbf{n}\rvert = \sqrt{3^2 + (-1)^2 + 2^2} = \sqrt{14}$	
$\lvert\mathbf{d}\rvert = \sqrt{(-1)^2 + (-5)^2 + 3^2} = \sqrt{35}$	
$8 = \sqrt{14}\sqrt{35}\cos\theta$	M1
$\theta = 68.8°$	
Angle between plane and line = 90° – 68.8°	dM1
= 21.2°	A1

(c) Find the coordinates of the point X.

(4)

Line: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7-\lambda \\ 3-5\lambda \\ -2+3\lambda \end{pmatrix}$

Substitute into plane: $3x - y + 2z = 10$

$3(7 - \lambda) - 1(3 - 5\lambda) + 2(-2 + 3\lambda) = 10$

$21 - 3\lambda - 3 + 5\lambda - 4 + 6\lambda = 10$

$14 + 8\lambda = 10$

$\lambda = -0.5$ M1

A1

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7-(-0.5) \\ 3-5(-0.5) \\ -2+3(-0.5) \end{pmatrix}$

$= \begin{pmatrix} 7.5 \\ 5.5 \\ -3.5 \end{pmatrix}$ dM1

coordinates of $X(7.5, 5.5, -3.5)$ A1

Question 3

Tyler invested a total of £5000 across three different accounts; a savings account, a property bond account and a share dealing account.

Tyler invested £400 more in the property bond account than in the savings account.

After one year

- the savings account had increased in value by 1.5%,
- the property bond account had increased in value by 3.5%,
- the share dealing account had **decreased** in value by 2.5%,
- the total value across Tyler's three accounts had increased by £79.

Form and solve a matrix equation to find out how much money was invested by Tyler in each account.

(7)

x = value of savings account
y = value of property bond account
z = value of share dealing account

$x + y + z = 5000$
$x + 400 = y$ so $x - y = -400$
$0.015x + 0.035y - 0.025z = 79$ M1

A1

These can be put into the matrix equation:

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0.015 & 0.035 & -0.025 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5000 \\ -400 \\ 79 \end{pmatrix}$$ M1

A1

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0.015 & 0.035 & -0.025 \end{pmatrix}^{-1}\begin{pmatrix} 5000 \\ -400 \\ 79 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0.25 & 0.6 & 10 \\ 0.25 & -0.4 & 10 \\ 0.5 & -0.2 & -20 \end{pmatrix}\begin{pmatrix} 5000 \\ -400 \\ 79 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1800 \\ 2200 \\ 1000 \end{pmatrix}$$ dM1

A1

Tyler invested £1800 in the savings account,
£2200 in the property bond account and
£1000 in the share dealing account. A1

Note: Candidates are expected to have a suitable calculator that will perform calculations with matrices up to at least a 3×3 matrix.

Question 4

The cubic equation

$$x^3 + 3x^2 - 8x + 6 = 0$$

has roots α, β and γ.

Without solving the equation, find the cubic equation whose roots are $(\alpha - 1)$, $(\beta - 1)$ and $(\gamma - 1)$, giving your answer in the form $w^3 + pw^2 + qw + r = 0$, where p, q and r are integers to be found.

(5)

$x^3 + 3x^2 - 8x + 6 = 0$	
If $\alpha \to \alpha - 1$ etc, then $x - 1 = w$, i.e. $x = w + 1$	B1
$(w + 1)^3 + 3(w + 1)^2 - 8(w + 1) + 6 = 0$	M1
$w^3 + 3w^2 + 3w + 1 + 3w^2 + 6w + 3 - 8w - 8 + 6 = 0$	
$w^3 + 6w^2 + w + 2 = 0$	dM1
	A1
	A1

Alternative:

for x:	
sum of roots: $\alpha + \beta + \gamma = -3$	
pair sums: $\alpha\beta + \beta\gamma + \alpha\gamma = -8$	
product: $\alpha\beta\gamma = -6$	B1
for w:	
sum of roots $= (\alpha - 1) + (\beta - 1) + (\gamma - 1)$	
$= \alpha + \beta + \gamma - 3$	
$= -3 - 3 = -6$	
pair sum $= (\alpha - 1)(\beta - 1) + (\alpha - 1)(\gamma - 1) + (\beta - 1)(\gamma - 1)$	
$= \alpha\beta + \alpha\gamma + \beta\gamma - 2(\alpha + \beta + \gamma) + 3$	
$= -8 - 2(-3) + 3 = 1$	
product $= (\alpha - 1)(\beta - 1)(\gamma - 1)$	
$= \alpha\beta\gamma - (\alpha\beta + \alpha\gamma + \beta\gamma) + (\alpha + \beta + \gamma) - 1$	
$= -6 - (-8) - 3 - 1 = -2$	M1
$w^3 + 6w^2 + w + 2 = 0$	dM1
	A1
	A1

Question 5

$$\mathbf{M} = \begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$$

(a) Show that **M** is non-singular.

(2)

det $\mathbf{M} = 1 \times 1 - \sqrt{3}(-\sqrt{3})$ M1
det $\mathbf{M} = 4$
so **M** is non-singular because det $\mathbf{M} \neq 0$ A1

The hexagon R is transformed to the hexagon S by the transformation represented by the matrix **M**.

Given that the area of hexagon R is 5 square units,

(b) find the area of hexagon S.

(1)

Area(S) = |det **M**| × Area(R)
$= 4 \times 5 = 20$ B1

The matrix **M** represents an enlargement, with centre (0, 0) and scale factor k, where $k > 0$, followed by a rotation anti-clockwise through an angle θ about (0, 0).

(c) Find the value of k.

(2)

$k = \sqrt{(\det \mathbf{M})} = \sqrt{4}$ M1
$k = 2$ A1

(d) Find the value of θ.

(2)

From the formula book:

Anticlockwise rotation of θ about O has transformation matrix $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Since the enlargement has scale factor 2, the rotation matrix is

$$\frac{1}{2} \times \begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$

so $\cos\theta = \frac{1}{2}$, $\sin\theta = \frac{\sqrt{3}}{2}$ M1

$\theta = 60°$ or $\frac{\pi}{3}$ radians A1

Question 6

(a) Prove by induction that for all positive integers n,

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

(6)

When $n = 1$, $\sum_{r=1}^{n} r^2 = 1$ and $\frac{1}{6}n(n+1)(2n+1) = \frac{1}{6}(1)(2)(3) = 1$, so true. B1

Assume general statement is true for $n = k$
so assume $\sum_{r=1}^{n} r^2 = \frac{1}{6}k(k+1)(2k+1)$ is true. M1

$\sum_{r=1}^{n} r^2 = \frac{1}{6}k(k+1)(2k+1) + (k+1)^2$ M1

$= \frac{1}{6}(k+1)[k(2k+1) + 6(k+1)]$

$= \frac{1}{6}(k+1)(2k^2 + 7k + 6)$ A1

$= \frac{1}{6}(k+1)(k+2)(2k+3)$

$= \frac{1}{6}(k+1)(\{k+1\}+1)(2\{k+1\}+1)$ A1

So the general result is true for $n = k + 1$.
As the general result has been shown to be true for $n = 1$,
then the general result is true for all positive integers. A1

(b) Use the standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r$ to show that for all positive integers n,

$$\sum_{r=1}^{n} r^2 = r(r+6)(r-6) = \frac{1}{4}n(n+1)(n-8)(n+9).$$

(4)

From the formula book: $\sum_{r=1}^{n} r^3 = \frac{1}{4} n^2(n+1)^2$

$$\sum_{r=1}^{n} r(r+6)(r-6) = \sum_{r=1}^{n}(r^3 - 36r)$$

$$= \sum_{r=1}^{n} r^3 - 36\sum_{r=1}^{n} r$$

$$= \frac{1}{4}n^2(n+1)^2 - 36 \times \frac{1}{2}n(n+1)$$ M1

A1

$$= \frac{1}{4}n(n+1)[n(n+1) - 72]$$ dM1

$$= \frac{1}{4}n(n+1)(n-8)(n+9)$$ A1

(c) Hence find the value of n that satisfies

$$\sum_{r=1}^{n} r(r+6)(r-6) = 17\sum_{r=1}^{n} r^2 .$$

(5)

$\frac{1}{4}n(n+1)(n-8)(n+9) = 17 \times \frac{1}{6} n(n+1)(2n+1)$ M1

$3(n-8)(n+9) = 34(2n+1)$ M1

$3n^2 + 3n - 216 = 68n + 34$

$3n^2 - 65n - 250 = 0$ A1

$(3n+10)(n-25) = 0$ M1

As n must be a positive integer, $n = 25$ A1

Question 7

Diagrams not drawn to scale

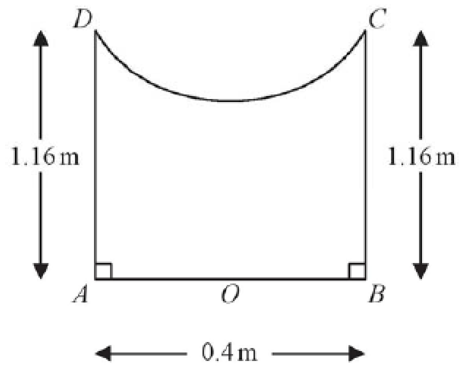


Figure 1

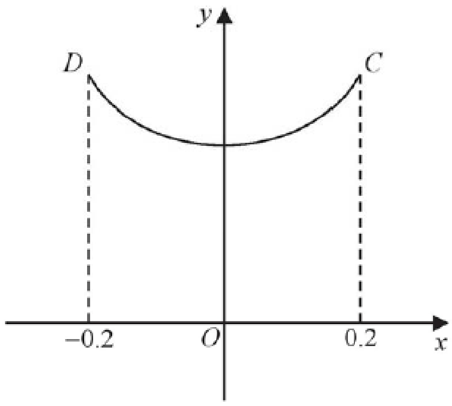


Figure 2

Figure 1 shows the central cross-section *AOBCD* of a circular birdbath, which is made of concrete. Measurements of the height and diameter of the birdbath, and the depth of the bowl of the birdbath have been taken in order to estimate the amount of concrete that was required to make this birdbath.

Using these measurements, the cross-sectional curve *CD*, shown in Figure 2, is modelled as a curve with equation

$$y = 1 + kx^2, \ -0.2 \le x \le 0.2,$$

where k is a constant and where O is the fixed origin.

The height of the bird bath measured 1.16 m and the diameter, *AB*, of the base of the birdbath measured 0.40 m, as shown in Figure 1.

(a) Suggest the maximum depth of the birdbath.

(1)

Depth = 1.16 – 1 = 0.16 m B1

(b) Find the value of k.

(2)

$y = 1 + kx^2$

$1.16 = 1 + k(0.2)^2$

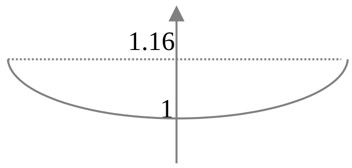
$k = \dfrac{0.16}{0.04}$ M1

$k = 4$ A1

(c) Hence find the volume of concrete that was required to make the birdbath according to this model. Give your answer, in m^3, correct to 3 significant figures.

(7)

To find the volume of the birdbath, first find the volume created when the area enclosed between the curve $y = 1 + 4x^2$ and the line $y = 1.16$ is rotated around the y-axis.



$V = \pi \int x^2 \, dy$

$y = 1 + 4x^2$ so $x^2 = \frac{1}{4}(y - 1)$

Volume = $\frac{1}{4}\pi \int_1^{1.16} y - 1 \, dy$ B1

(limits 1.16 and 1) M1

$= \frac{1}{4}\pi \left[\frac{1}{2}y^2 - y\right]_1^{1.16}$ M1

A1

substitute in limits

$= \frac{1}{4}\pi\,[(\frac{1}{2}(1.16)^2 - 1.16) - (\frac{1}{2} - 1)]$

$= 0.0032\pi$

Then subtract this answer from the volume of a cylinder with radius 0.2 and height 1.16.

Cylinder volume $= \pi\,(0.2)^2(1.16)$ B1

$= 0.0464\pi$

Birdbath volume $= 0.0464\pi - 0.0032\pi$ ddM1

$= 0.0432\pi$

$= 0.1357168...$

$= 0.136 \text{ m}^3$ A1

(d) State a limitation of the model.

(1)

e.g. The measurements may not be accurate.
The inside surface of the bowl may not be smooth.
There may be wastage of concrete when making the birdbath. B1

It was later discovered that the volume of concrete used to make the birdbath was 0.127 m^3 correct to 3 significant figures.

(e) Using this information and the answer to part (*c*), evaluate the model, explaining your reasoning.

(1)

e.g. % error = $\left(\frac{0.136 - 0.127}{0.127}\right) \times 100\% = 7.0866...\%$
so not a good estimate because the volume of concrete needed to make the birdbath is approximately 7% lower than that predicted by the model.

e.g. we might expect the actual amount of concrete to exceed that which the model predicts due to wastage, so the model does not look suitable since it predicts more concrete than was used. B1

Question 8

(a) Shade on an Argand diagram the set of points

$$\{z \in \mathbb{C}: |z - 4\mathrm{i}| \leq 3\} \cap \{ z \in \mathbb{C}: -\frac{\pi}{2} < \arg(z + 3 - 4\mathrm{i}) \leq \frac{\pi}{4} \}.$$

(6)

$|z - 4\mathrm{i}| \leq 3$

This is a circle — M1

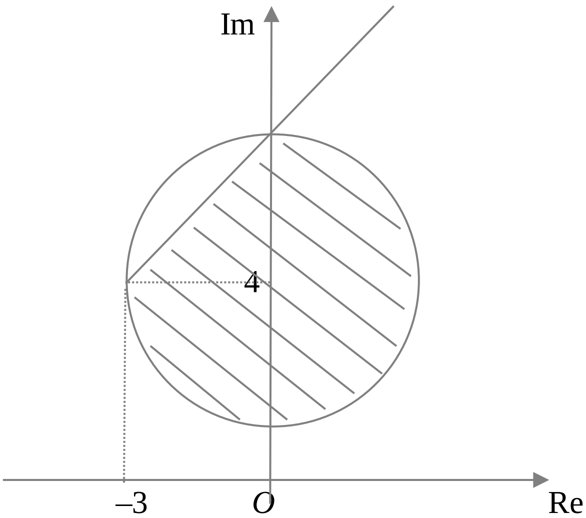
centre (0, 4) radius 3. — A1

$-\frac{\pi}{2} < \arg(z - (-3 + 4\mathrm{i})) \leq \frac{\pi}{4}$

This is a half line — M1

starting from (–3, 4) and intersects the top of the circle on the *y*-axis. — A1

The region required is inside the circle and below the half line. — ddM1



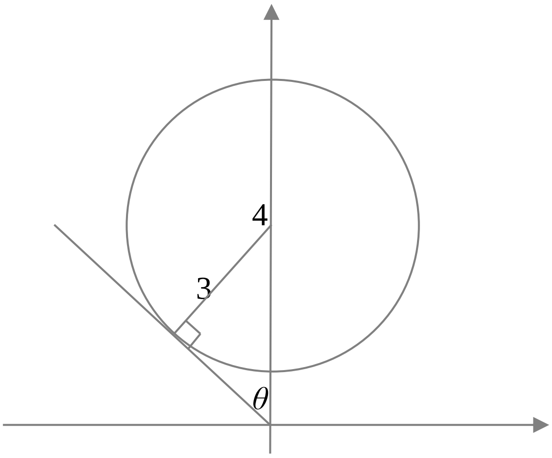
A1

The complex number w satisfies $|w - 4\mathrm{i}| = 3$.

(b) Find the maximum value of arg w in the interval $(-\pi, \pi]$.
Give your answer in radians correct to 2 decimal places.

(2)

The maximum argument is at the point where the line touches the circle as a tangent as shown.



max. arg $w = \dfrac{\pi}{2} + \theta$ where $\sin\theta = \dfrac{3}{4}$ M1

$= 2.42$ rad (2dp) A1

Question 9

An octopus is able to catch any fish that swim within a distance of 2 m from the octopus's position.

A fish *F* swims from a point *A* to a point *B*.

The octopus is modelled as a fixed particle at the origin *O*.

Fish *F* is modelled as a particle moving in a straight line from *A* to *B*.

Relative to *O*, the coordinates of *A* are (–3, 1, –7) and the coordinates of *B* are (9, 4, 11), where the unit of distance is metres.

(a) Use the model to determine whether or not the octopus is able to catch fish *F*.

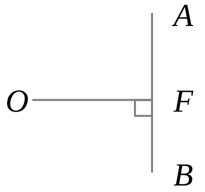
(7)

$$\overrightarrow{AB} = \begin{pmatrix} 9 \\ 4 \\ 11 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$$ M1

Line: $\mathbf{r} = \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} \quad (= \overrightarrow{OF})$ M1

The shortest distance from *O* to line *AB* will be where the scalar product

$$\overrightarrow{OF} \cdot \overrightarrow{AB} = 0$$



$$\begin{pmatrix} -3+12\lambda \\ 1+3\lambda \\ -7+18\lambda \end{pmatrix} \bullet \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} = 0$$ dM1

$-36 + 144\lambda + 3 + 9\lambda - 126 + 324\lambda = 0$

$477\lambda - 159 = 0$

$\lambda = \frac{1}{3}$ A1

$$\overrightarrow{OF} = \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

Minimum distance $|\overrightarrow{OF}| = \sqrt{1^2 + 2^2 + (-1)^2}$ dM1

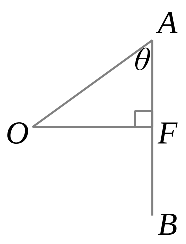
$= \sqrt{6} = 2.449...$ A1

This is >2, so the octopus is not able to catch the fish F. A1

Alternative 1

$$\overrightarrow{AB} = \begin{pmatrix} 9 \\ 4 \\ 11 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$$ M1

Find the acute angle θ from the scalar product formula.



$$\overrightarrow{OA} . \overrightarrow{AB} = \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \bullet \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$$ M1

$= -36 + 3 - 126$
$= -159$

$OA = \sqrt{(-3)^2 + 1^2 + (-7)^2} = \sqrt{59}$
$AB = \sqrt{12^2 + 3^2 + 18^2} = \sqrt{477}$

Using scalar product formula
$|-159| = \sqrt{59}\,\sqrt{477}\cos\theta$ dM1
$\theta = 18.596197...$ A1

Minimum distance $OF = \sqrt{59}\sin(18.596197...)$ dM1
$= \sqrt{6} = 2.449...$ A1
This is >2, so the octopus is not able to catch the fish F. A1

Alternative 2:

$$\overrightarrow{AB} = \begin{pmatrix} 9 \\ 4 \\ 11 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$$ M1

Line: $\mathbf{r} = \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$ $(= \vec{OF})$ M1

$|\overrightarrow{OF}|^2 = (-3 + 12\lambda)^2 + (1 + 3\lambda)^2 + (-7 + 18\lambda)^2$ dM1

$= 9 - 72\lambda + 144\lambda^2 + 1 + 6\lambda + 9\lambda^2 + 49 - 252\lambda + 324\lambda^2$
$= 477\lambda^2 - 318\lambda + 59$ A1

differentiate

$$\frac{d\,|\overrightarrow{OF}|^2}{d\lambda} = 954\lambda - 318$$

$$\frac{d\,|\overrightarrow{OF}|^2}{d\lambda} = 0 \text{ for minimum}$$

$954\lambda - 318 = 0$

$\lambda = \dfrac{1}{3}$

or complete the square

$$|\overrightarrow{OF}|^2 = 53(9\lambda^2 - 6\lambda) + 59$$

$$= 53[(3\lambda - 1)^2 - 1] + 59$$

$$= 53(3\lambda - 1)^2 - 53 + 59$$

$$= 53(3\lambda - 1)^2 + 6$$ dM1

Minimum $|\overrightarrow{OF}|^2 = 6$

Minimum distance = $\sqrt{6} = 2.449...$ A1

This is >2, so the octopus is not able to catch the fish *F*. A1

(b) Criticise the model in relation to fish *F*. **(1)**

e.g.

Fish *F* may not swim in an exact straight line from *A* to *B*.

Fish *F* may hit an obstacle whilst swimming from *A* to *B*.

Fish *F* may deviate his path to avoid being caught by the octopus. B1

(c) Criticise the model in relation to the octopus. **(1)**

e.g.

Octopus is effectively modelled as a particle – so we may need to look at where the octopus's mass is distributed.

Octopus may during fish *F* 's motion move away from its fixed location at *O*.

B1

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